Implicit/explicit spectral viscosity and large-scale SGS effects

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1 Introduction

Subgrid-scale (SGS) modelling based on regularization has become a popular approach for Large Eddy Simulation (LES). When the regularization is driven by the numerical error or by an extra discrete operator like a filter, it is usual to refer to implicit LES in the sense that the discretization provides an artificial dissipation interpreted as a substitute of SGS modelling. Typically, it is expected that this artificial dissipation is inactive at very large scales thanks to the numerical convergence of the associated discretization. This assumption of large-scale dynamics virtually free from any artificial dissipation can even be intentionally extended on a wide range of scales through an optimal design of the associated discrete schemes. This idea can also be recovered in explicit SGS models with for instance the concept of Spectral Vanishing Viscosity (SVV) [1] and the Variational Multiscale (VMS) methods [2].

The goal of this study is to assess this inviscid assumption at very large scales for a flow at high Reynolds number while using DNS results to estimate the exact energy transfers from large to SGS. These transfers are investigated in the challenging situation of a flow subjected to a complete transition up to a fully developed turbulent state. The corresponding benchmark is the Taylor-Green vortex problem at $Re = 20{,}000$. In a previous work [3], using an implicit SVV associated to the differentiation errors of the viscous term, we have shown that very accurate results can be obtained by LES at $Re = 10{,}000$ with a reduction of the number of degrees of freedom (DOF) of 8^3 by reference to DNS. In this study, we want to investigate the ability of this type of SGS modelling (without any direct influence on large

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scales) at higher Reynolds number and with a stronger reduction of DOF. Note that both Reynolds numbers $Re = (10{,}000; 20{,}000)$ correspond to fully turbulent conditions as suggested by the value of their counterparts based on the Taylor microscale $Re_\lambda \approx (200; 300)$ obtained after the complete turbulence breakdown at $t \approx 13$.

2 *A priori* analysis from DNS results

The DNS of reference as well as the LES are performed using the sixth-order flow solver "Incompact3d" which is kinetic energy conserving in the discrete and inviscid sense (up to the time advancement error). For the present high Reynolds number case $Re = 20{,}000$, 3456^3 mesh nodes are required for a computational domain of $(2\pi)^3$ but using some symmetries of the problem, the number of DOF is actually divided by 8. Here, the goal is to carry out counterpart LES where the number of DOF and computational cost are reduced by 16^3 and 16^4 respectively leading to a cutoff wavenumber of $k_c = 108$ for the LES mesh against 1728 for the DNS one.

Following our conclusions in a previous work, the targeted LES solution is defined using a progressive spatial filter as illustrated in figure 1-left where raw and filtered DNS energy spectra are compared. The filter is obtained by solving the Lin equation using a simplified spectral Pao-like closure while taking the implicit SGS dissipation into account [3]. An important remark is that this filter is applied once in each spatial direction and not in all directions as it would be for an isotropic filter. This 1D definition of the filter is believed to be more significant by reference to the actual anisotropy of the LES mesh. Using this specific filter applied on the DNS data, the time evolution of the supergrid scale kinetic energy $\bar{E}_k$ can be computed *a priori* with its associated total dissipation $\bar{\varepsilon} = -d\bar{E}_k/dt$. Then, it is easy to estimate the viscous large-scale dissipation ε_{LS} and its complementary SGS part ε_{SGS} such as $\bar{\varepsilon} = \varepsilon_{LS} + \varepsilon_{SGS}$. These dissipations as well as the full DNS dissipation are presented in figure 1-right where it can be seen that this benchmark is very challenging with SGS dissipation ε_{SGS} up to 90% of the total dissipation $\bar{\varepsilon}$, this unequal distribution giving a major role to the SGS model.

To have a more detailed view of the kinetic energy transfer, a scale by scale analysis of the SGS dissipation can be done starting from the large-scale Lin equation decomposed as

$$\left(\frac{\partial}{\partial t} + 2\nu k^2\right)\bar{E}(k,t) = \bar{T}(k,t) + T_{SGS}(k,t) \qquad (1)$$

where $\bar{E}(k,t)$ is the kinetic energy of the filtered solution, $\bar{T}(k,t)$ the transfer term involving only the filtered solution (i.e. explicitly computed in LES) and $T_{SGS}(k,t)$ the remaining term that describes transfers between the supergrid and subgrid scales. In this formalism expressed in the Fourier space, $T_{SGS}(k,t)$ is simply the spectral density of ε_{SGS} that leads to the introduction of the spectral eddy viscosity

$$\nu_t(k,t) = -\frac{T_{SGS}(k,t)}{2k^2\bar{E}(k,t)} \qquad (2)$$

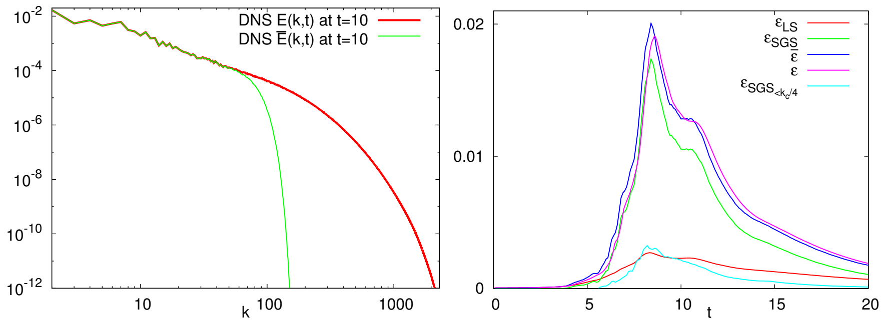

Fig. 1 Left: raw and filtered DNS energy spectra. Right: time evolution of the viscous large-scale, SGS, SGS for $k < k_c/4$, filtered DNS and full DNS dissipations (ε_{LS}, ε_{SGS}, $\varepsilon_{SGS_{<k_c/4}}$, $\bar{\varepsilon}$, ε).

Using the DNS data and following the procedure of [4] adapted in the present context, $\nu_t(k,t)$ is estimated from 200 snapshots distributed throughout the calculation.

Figure 2-left presents 5 samples of $\nu_t(k,t)$ with a normalization based on the molecular viscosity ν. This figure clearly exhibits the dominant transfers close to k_c meaning that the "hyperviscous feature" is observed for this benchmark, especially in the early transition. Figure 2-left also reveals that more distant triad interactions result in high values of $\nu_t(k,t)$ at small k, not only during the transition (for instance at $t = 10$) but also until the end of the simulation where a fully developed non-equilibrium turbulence is observed. Then, at least qualitatively, the "plateau-cusp" profile of the spectral eddy viscosity, as predicted by two-point closure theories at high Reynolds number [5, 6, 7], is well recovered in the present *a priori* analysis.

In figure 2-right, the average value $\nu_{t_{<k_c/4}}$ of $\nu_t(k,t)$ for $2 < k < k_c/4$ (as an estimation of the "plateau" value) and its cutoff value $\nu_{t_{=k_c}}$ at $k = k_c$ are plotted throughout the simulation. The values of $\nu_{t_{=k_c}}$ are found to be very high in the early transition but exhibit a global decrease as the turbulence develops. More importantly, the significance of direct effect of SGS on the large scale dynamics is confirmed during the turbulence breakdown (with $\nu_{t_{<k_c/4}}$ that can be more than 4 times larger than ν) but also when the turbulence is fully developed where $\nu_{t_{<k_c/4}}$ is still of the same order as ν. This observation is against the lack of any direct dissipative effect at large scales in the SGS modelling as it is assumed in implicit LES, SVV or VMS.

It could be thought that despite the high values of $\nu_t(k,t)$ for $k < k_c/4$, the corresponding fraction of SGS dissipation

$$\varepsilon_{SGS_{<k_c/4}} = 2\int_0^{k_c/4} \nu_t(k,t)k^2\bar{E}(k,t)\,dk \tag{3}$$

is negligible. In figure 1-right, it can be seen that this quantity can actually be about 20% of the full SGS dissipation ε_{SGS} during the turbulence breakdown while remaining about 10% until the end of the calculation. In the next section, it will be examined whether this significant contribution can be ignored in practical LES.

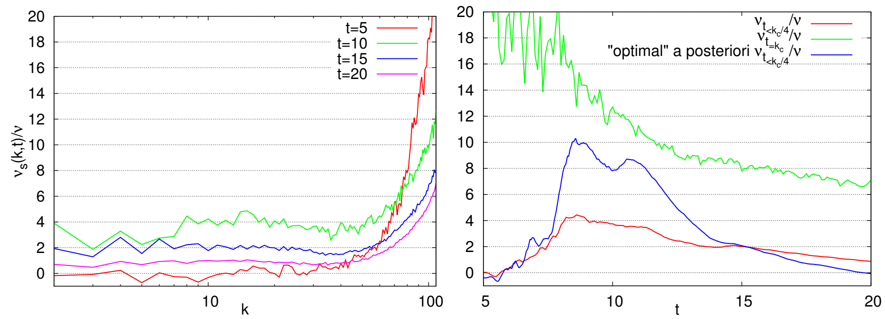

Fig. 2 Left: spectral eddy viscosity $\nu_t(k,t)$ at $t = 5, 10, 15$ and 20. Right: time evolution of the average $\nu_{t_{<k_c/4}}$ and cutoff $\nu_{t_{=k_c}}$ values of $\nu_t(k,t)$.

3 *A posteriori* analysis of LES results

The same flow configuration is investigated by LES (i) without any SGS modelling; (ii) with the standard/dynamic Smagorinsky model and (iii) with our implicit SVV [3]. Figure 3 presents the time evolution of the total dissipation $\bar{\varepsilon}$ obtained for the different LES. The very unrealistic behaviour observed for the no-model case confirms the major role of the SGS modelling for the present high Reynolds number case where the LES are based on a coarse mesh by comparison to DNS. The strong overestimation of $\bar{\varepsilon}$ without SGS model corresponds to an almost complete thermalization of the flow due to the development of small-scale spurious oscillations during the transition, as it can be clearly observed by visualization and spectral analysis (not shown for conciseness). The standard and dynamic Smagorinsky models are found to lead to a partial thermalization resulting in an overdissipative behaviour in the early transition. The resulting damping of small-scale spurious oscillations has, in a second stage, a feedback effect with an underestimation of $\bar{\varepsilon}$.

The use of implicit SVV prevents any thermalization with a good prediction of $\bar{\varepsilon}$ in the early transition. However, the excellent agreement obtained in [3] at $Re = 10{,}000$ with a less coarse mesh is not recovered for the present more demanding benchmark. In particular, the peak of dissipation cannot be captured (underestimation of $\bar{\varepsilon}$) and as a subsequent feedback effect, a spurious secondary peak (overestimation of $\bar{\varepsilon}$) can be clearly observed. A similar spurious secondary peak can be observed for the enstrophy ζ (see figure 4-right).

For the implicit SVV, the main discrepancy can be attributed to the poor reproduction of the main peak of $\bar{\varepsilon}$ that has the potential to spoil the flow any time thereafter. Even a very strong increase of the implicit SVV near k_c is unable to capture this peak (not shown for conciseness). A spectral analysis shows that the kinetic energy is overestimated at large scales during this particular moment, especially in the range $10 < k < k_c/2$, as illustrated in figure 4-left. This overestimation is interpreted as the consequence of the quasi-inviscid cascade at large scales. For this type of LES, free from distant triad interaction modelling, the overestimation of $\bar{E}(k,t)$ in

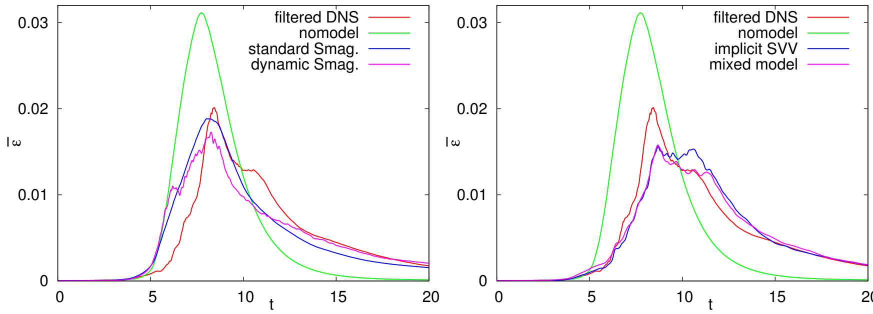

Fig. 3 Time evolution of the total dissipation $\bar{\varepsilon}$ predicted by LES.

the range $10 < k < k_c/2$ can be interpreted as a bottleneck effect. This interpretation is consistent with the *a priori* analysis presented in previous section. This view can even be supported quantitatively by observing that the main peak of dissipation is underestimated by about 22%. Because the implicit SVV is essentially inactive for $k < k_c/4$, this behaviour may be related to the *a priori* ratio $\varepsilon_{SGS_{<k_c/4}}/\varepsilon_{SGS} \approx 20\%$ reported in the previous section.

To restore the peak, an idea could be to combine an explicit model with our implicit SVV with the hope that the former can boost the total dissipation during the transition while the latter can avoid the unrealistic partial thermalization. This kind of mixed model approach has been tried with the standard Smagorinsky (see figure 3-right) but without any improvement for the prediction of the main peak of $\bar{\varepsilon}$. However, it is worth noting that this mixed model can remove the spurious secondary peak.

Another attempt was to modify our implicit model to allow non-vanishing spectral viscosity while adjusting its "plateau" value in order to follow precisely the time evolution of $\bar{\varepsilon}$. Such LES can be considered as "optimal" in terms of ability to predict the filtered kinetic energy $\bar{E}_k$. The time evolution of the resulting plateau value is presented in figure 2-right. Even if significantly higher levels of eddy viscosity are required by comparison with the *a priori* estimation of $\nu_{t_{<k_c/4}}$, it confirms the ability of a direct influence of the SGS model on very large scales to ensure the correct energy dissipation. However, this "optimal" approach (that is only a test case in the sense that it requires to know the expected time evolution of $\bar{E}_k$) is found to underestimate $\bar{E}(k,t)$ at small scales (see figure 4-left) with a resulting strong underestimation of enstrophy (see figure 4-right).

4 Conclusion

In order to investigate the scale-selective influence of SGS on the large scale dynamics, DNS and LES are performed for the Taylor-Green vortex problem. An a priori analysis confirms the interest of the hyperviscous feature at small scale as used in

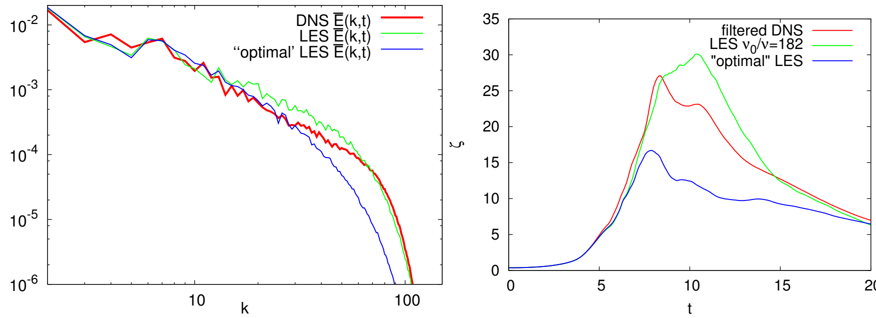

Fig. 4 Left: energy spectra $\bar{E}(k,t)$ at $t = 10$. Right: time evolution of the enstrophy ζ.

implicit LES, SVV and VMS. However, the assumption of zero SGS dissipation at very large scales is found unrealistic for the high Reynolds number and coarse LES mesh considered. A posteriori analysis shows that SGS modelling based on the assumption of an inviscid cascade leads to a bottleneck effect on the kinetic energy spectrum with a significant underprediction of the total SGS dissipation. The simple addition of a constant eddy viscosity, even targeted to be optimal in terms of SGS dissipation, is unable to give realistic results. To allow accurate predictions by LES, a specific closure that incorporates both the hyperviscous feature (i.e. regularisation) and the expected SGS dissipation at large scales has to be developed.

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